18. Yu. G. Rykov, "Behavior over time of generalized degenerate parabolic equations with nonisotropic nonexponential nonlinearities," in: Differential Equations and Their Application [in Russian], Izd. Mosk. Gos. Univ., Moscow (1984).
19. S. N. Antontsev, "Localization of the solutions of certain degenerate equations of continuum mechanics," in: Problems of Mathematics and Mechanics [in Russian], Nauka, Novosibirsk (1983).
20. S. N. Antontsev and A. A. Panin, "Localization of the solutions of the equations of a viscous gas with viscosity dependent on density," Din. Sploshnoi Sredy, 86 (1988).
21. S. N. Antontsev, "Localization of the solutions of degenerate nonlinear elliptic equations," in: Equations in Partial Derivatives and Problems with a Free Boundary [in Russian], Naukova Dumka, Kiev (1983).
22. S. N. Antontsev, A. V. Kazhikhov, and V. N. Monakhov, Boundary-Value Problems of the Mechanics of Inhomogeneous Fluids [in Russian], Nauka, Novosibirsk (1983).
23. S. N. Antontsev, G. P. Epikhov, and A. A. Kashevarov, Systematic Mathematical Modeling of Water Transport Processes [in Russian], Nauka, Novosibirsk (1986).
24. L. V. Ovsyannikov, Introduction to Continuum Mechanics [in Russian], Vol. 2, Novosibirsk Gos. Univ. (1977).
25. A. Kh. Mirzadzhanzade and P. M. Ogibalov, Transient Motions of Viscoplastic Media, Izd. Mosk. Gos. Univ., Moscow (1970).
26. P. P. Mosolov and V. P. Myasnikov, Mechanics of Rigid-Plastic Media [in Russian], Nauka, Moscow (1981).
27. M. Bohm, "On a nonhomogeneous non-Newtonian fluid," Rep./Lefschetz Center for Dynam. Systems. Division Appl. Mat., Brown Univ., Providence (1983), pp. 83-8.
28. J. L. Lions, Some Methods of Solving Nonlinear Boundary-Value Problems [Russian translation], Mir, Moscow (1972).
29. O. A. Ladyzhenskaya, Mathematical Aspects of the Dynamics of a Viscous Incompressible Fluid [in Russian], Nauka, Moscow (1970).
30. S. N. Antontsev, "Metastable localization of the solutions of degenerate parabolic equations of general form," Din. Sploshnoi Sredy, 83 (1987).
31. J. I. Diaz, Nonlinear Partial Differential Equations and Free Boundaries, Vol. 1, Elliptic Equations, Putnam (1985) (Putnam Res. Not. Math., 106).

STABILITY OF REGULAR SHOCK WAVE REFLECTION

V. M. Teshukov

UDC 533.6.011

As is known, the problem of steady supersonic inviscid gas flow around an infinite wedge has a nonunique solution [1]. One of the solutions determines the flow with a weak attached compression shock, and the other with a strong shock. An analogous nonuniqueness occurs in the problem of regular reflection of an oblique compression shock from a rigid wall (strong and weak reflected shocks). Stability of the flow with weak and strong reflected shocks relative to small nonstationary perturbations is investigated in this paper. Correctness of the problem of the perturbations of the flow with a weak reflected shock and incorrectness of the problem of perturbations of the flow with the strong shock are established. This result determines the stability boundary of regular shock reflection. Questions of the stability of flows with strong and weak shocks have long attracted the attention of researchers [2]. Analytic results were obtained earlier just for model simplified formulations of the gas dynamic perturbation problem [3-5]. Assertions about the stability of flows with weak shocks and the instability of flows with strong shocks were expressed in [5, 6] in connection with an analysis of the results of calculation experiments.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 26-33, March-April, 1989. Original article submitted August 1, 1988.

1. FORMULATION OF THE PROBLEM

Let us consider the steady flow of an inviscid nonheat-conducting gas that occurs during reflection of an oblique compression shock from a rigid wall. Let Γ_0 be the incident compression shock (Fig. 1), Γ_1 the reflected compression shock, and $y = 0$ correspond to the rigid wall. The densities ρ_i, pressures p_i, velocity vectors $\mathbf{u}_i = (u_i, v_i)$ ($i = 0, 1, 2$; $v_0 = v_2 = 0$) in the domains 0, 1, 2 are constant, $p_2 > p_1 > p_0$. These quantities are connected by the Hugoniot relations on the fronts Γ_0, Γ_1.

According to the given incident shock parameters (quantities with subscripts 0, 1) the main quantities in the domain 2 can be found by the shock polar method. Let q, ψ be polar coordinated in the hodograph plane $u = q\cos\psi$, $v = q\sin\psi$. The shock polar equation

$$\psi - \psi_i = \pm \arcsin\left[\frac{(p-p_i)(\tau_i - \tau - \tau_i^2 q_i^{-2}(p-p_i))}{q_i^2 - (p-p_i)(\tau_i + \tau)}\right]^{1/2} \tag{1.1}$$

follows from the Hugoniot relationships on the shock. Let $\tau = \rho^{-1}$; $\tau = \tau(p, \tau_i, p_i)$ by virtue of the Hugoniot adiabat equation; the quantities without subscript correspond to the flow behind the shock and those with the subscript ahead of the shock. Conditions assuring uniqueness of the transition point through the speed of sound on the shock polar [points where the equality $q^2 = q_i^2 - (p - p_i)(\tau_i + \tau) = c^2$ is achieved, and c is the speed of sound] and the presence of just two points of intersection of the shock polar with the lines $\psi = \text{const}$ ($|\psi - \psi_i| < \psi_m$, ψ_m is the limit angle of rotation of the velocity vector in the oblique shock) are obtained in [7] for the equations of state of a normal gas. We consider these conditions satisfied. The shock polar configuration corresponding to regular oblique shock reflection from a wall is displayed in Fig. 2. The flow in domain 2 is determined nonuniquely: the point 2 corresponds to flow behind a weak reflected shock, and 2* to a point behind a strong shock ($p_2^* > p_2$).

It is also shown in [7] that in gases with the equations of state $\varepsilon = \varepsilon(\tau, p)$, $p = g(\tau, S)$ (S is the entropy and ε the internal energy) satisfying the condition

$$(p + \tau g_\tau)\varepsilon_p + p\tau \leqslant 0, \tag{1.2}$$

behind a strong reflected shock, the flow is always subsonic [a polytropic gas satisfies (1.2)]. If condition (1.2) is not satisfied, then the flow behind the strong shock can be both sub- and supersonic. Nonstationary perturbations of subsonic flows behind both weak and strong reflected compression shocks are examined in this paper.

The stability of stationary shocks relative to nonstationary perturbations is studied in [8, 9]. Initial perturbations whose support lies in a band of finite width adjoining the shock front are examined in [9]. It is established that if the main flow parameters satisfy the inequalities

$$-1 < \Delta < \frac{1 - M_n^2 - RM_n^2}{1 - M_n^2 + RM_n^2} \quad \left(R = \frac{\tau_i}{\tau},\right.$$
$$\left.\Delta = \frac{p - p_i}{\tau_i - \tau}\frac{\partial\tau(p, \tau_i, p_i)}{\partial p}, \quad M_n = \frac{u_n}{c}, \quad u_n = \mathbf{u}\cdot\mathbf{n}\right), \tag{1.3}$$

then the perturbations damp out on the shock front as time passes. If

$$\frac{1 - M_n^2 - RM_n^2}{1 - M_n^2 + RM_n^2} < \Delta < 1 + 2M_n, \tag{1.4}$$

then the perturbations oscillate and do not damp out with the lapse of time. For $\Delta > 1 + 2M_n$ or $\Delta < -1$ an instability in a linear approximation holds.

We shall consider that conditions (1.3) or (1.4) are satisfied on the incident and reflected compression shocks. A description of the classes of equations of state is obtained in [10] for which either the inequalities (1.3) or (1.4) are always satisfied on the shocks. In particular, (1.3) are satisfied on any shock front if and only if the equations of state satisfy condition (1.2). Later the domain of the parameters (1.3) will be called the strong stability domain while the domain of the parameters (1.4) is the neutral stability domain. It should be noted that the stability conditions (1.3) and (1.4) still do not guarantee correctness of the formulation of the perturbation problem in domains 0, 1, 2. Examples of

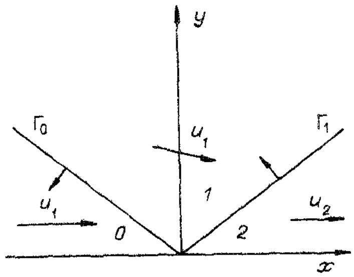

Fig. 1

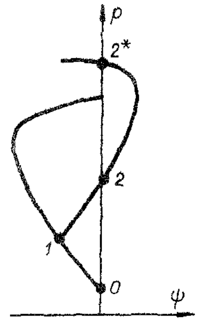

Fig. 2

formulations of problems in angular domains are known for hyperbolic systems of equations when the boundary conditions determine the correct problem locally in the neighborhood of each face, but the problem in an angular domain as a whole is incorrect.

Let nonstationary main flow changes occur because of the introduction of perturbations at the time t = 0 in the domain 0. By virtue of the supersonic nature of the flow the influence of these perturbations are propagated into domains 1 and 2 for t > 0. Construction of the perturbed solution in domain 0 reduces to solving the correct mixed problem with Cauchy data at t = 0 and linearized Hugoniot conditions on Γ_0 (this problem was considered in [9, 11]). After the perturbations have been constructed in domains 0 and 1, there remains to find the perturbations in domain 2.

We represent the flow parameters $\bar{u}$, $\bar{v}$, $\bar{p}$, $\bar{S}$ in domain 2 in the form

$$\bar{p} = p_2 + \rho_2 c_2^2 p, \quad \bar{u} = u_2 + c_2 u, \quad \bar{v} = c_2 v, \quad \bar{S} = S_2(1 + S)$$

(u, v, p, S are dimensionless small perturbations). These functions satisfy the linearized gas dynamics equations

$$\begin{gathered} u_t + \mathrm{M}u_x + p_x = 0, \; v_t + \mathrm{M}v_x + p_y = 0, \\ p_t + \mathrm{M}p_x + u_x + v_y = 0, \quad S_t + \mathrm{M}S_x = 0 \quad (\mathrm{M} = u_2 c_2^{-1}) \end{gathered} \tag{1.5}$$

(the Cartesian coordinates x, y, and $t = \bar{t}c_2$, $\bar{t}$ is the time selected as independent variables). We write the equation of the perturbed reflected shock front in the form

$$x = ay + \Phi(y, t) \quad \left(a = \left(\frac{u_2^2(\tau_1 - \tau_2)}{\tau_2^2(p_2 - p_1)} - 1\right)^{1/2}\right).$$

Here $x = ay$ yields Γ_1, while $\Phi(y, t)$ is the desired small perturbation. The initial-boundary value conditions of the mixed problem governing the perturbations in domain 2 ($x \geq y$, $y \geq 0$, $t \geq 0$) have the form

$$\begin{gathered} t = 0\colon u = v = p = S = 0;\; y = 0\colon v = 0; \\ x = ay\colon av - u = (1 + a^2)(1 - \Delta)(2\mathrm{M})^{-1}p + f_1(y, t), \\ au + v = \mathrm{M}(R - 1)(1 + a^2)^{-1}\Phi_y + f_2(y, t), \\ p = 2\mathrm{M}(R^{-1} - 1)(1 + a^2)^{-1}(1 + \Delta)^{-1}(\Phi_t + a\mathrm{M}(1 + a^2)^{-1}\Phi_y) + \\ + f_3(y, t), \\ S = \rho_2 c_2^2\left(1 - g_\tau(\tau_2, S_2)\frac{\partial\tau}{\partial p}(p_2, \tau_1, p_1)\right)(S_2 g_S(\tau_2, S_2))^{-1}p + f_4(y, t); \\ x = y = 0\colon \Phi(0, t) = x(t). \end{gathered} \tag{1.6}$$

The first group of conditions are Cauchy data, then the condition of nonpenetration at y = 0 follows, then the linearized Hugoniot conditions on Γ_1, and the last is the condition for reflected shock front passage through the point of perturbed incident front incidence. Here $f_i(x, t)$, $x(t)$ are given functions expressed in terms of known perturbed flow parameters in domain 1.

Let us determine the domain of main flow parameters corresponding to weak and strong reflected shocks. At the point 2 (see Fig. 2), $d\psi/dp > 0$, while $d\psi/dp < 0$ at point 2*. Evaluation of $d\psi/dp$ by using (1.1) results in the expression $(d\psi/dp)_2 = (a^2(1 - \Delta) - R(1 + \Delta))(2\rho_2 c_2{}^2 a M^2)^{-1}$. Consequently, the parameter domain described by the inequality $a^2(1 - \Delta) - R(1 + \Delta) > 0$ corresponds to the weak reflected shock while the domain described by the reverse inequality corresponds to the strong reflected shock. Since

$$\frac{a^2 - R}{a^2 + R} - \frac{1 - M_n^2 - RM_n^2}{1 - M_n^2 + RM_n^2} = \frac{2R(M^2 - 1)}{(a^2 + R)(1 - M_n^2 + RM_n^2)} < 0$$

for $M < 1$, then the strong stability conditions (1.3) are certainly satisfied on the weak shock. Strong stability conditions (1.3) are satisfied on the strong shock for

$$\frac{a^2 - R}{a^2 + R} < \Delta < \frac{1 - M_n^2 - RM_n^2}{1 - M_n^2 + RM_n^2}$$

and neutral stability conditions in the remaining domain of parameters. Therefore, the investigation of regular shock reflection stability reduces to the problem (1.5), (1.6). Let us note that study of nonstationary perturbations in the domain behind an attached compression shock during flow around an infinite wedge reduces to solving exactly the same problem. Consequently, the results of investigating problem (1.5), (1.6) are completely applicable to an analysis of the stability of supersonic gas flow around a wedge.

2. CONSTRUCTION OF THE SOLUTION

It is convenient to introduce polar coordinates $x = r\cos\theta$, $(1 - M^2)^{1/2} y = r\sin\theta$ in domain 2. Domain 2 in the new variables corresponds to the half-strip $0 \le r < \infty$, $0 \le \theta \le \theta_0$ $[\sin\theta_0 = (1 - M^2)^{1/2}(a^2 + 1 - M^2)^{-1/2}$, $\cos\theta_0 = a(a^2 + 1 - M^2)^{-1/2}]$.

Let us consider the Laplace transform of the vector-function $\mathbf{U} = (u, v, p)$ with respect to time

$$\mathbf{U}_1(\lambda, r, \theta) = \int_0^\infty e^{-\lambda t}\mathbf{U}(t, r, \theta)\,dt.$$

Let us also introduce the Laplace transform of the vector $\mathbf{U}_1$ with respect to the radius

$$\mathbf{U}_2(\lambda, z, \theta) = \int_0^\infty e^{-zr}\mathbf{U}_1(\lambda, r, \theta)\,dr.$$

The requirement of quadratic summability of $|\mathbf{U}_1(\lambda, r, \theta)|$ in r assures analyticity of $\mathbf{U}_2(\lambda, z, \theta)$ for positive $\operatorname{Re}\lambda$ in the domain $\operatorname{Re} z > 0$. To construct the solution of the problem (1.5), (1.6), we introduce the new desired functions $\eta(\lambda, \mu, \theta)$, $\sigma(\lambda, \mu, \theta)$, $L(\lambda, \mu, \theta)$ $[\mu = z + kM\cos\theta$, $k = \lambda(M^2 - 1)^{-1}]$:

$$\begin{aligned} 2\eta &= (\mu^2 - k^2)^{1/2}P_3 + U_3 ki\sin\theta - V_3 i(k\cos\theta - M\mu)(1 - M^2)^{-1/2},\\ 2\sigma &= (\mu^2 - k^2)^{1/2}P_3 - U_3 ki\sin\theta + V_3 i(k\cos\theta - M\mu)(1 - M^2)^{-1/2},\\ L &= (kM\cos\theta - \mu)P_3 + U_3(k\cos\theta - M\mu) + k\sin\theta(1 - M^2)^{1/2}V_3 \end{aligned} \quad (2.1)$$

$[\mathbf{U}_3(\lambda, \mu, \theta) = (U_3, V_3, P_3) = \mathbf{U}_2(\lambda, \mu - kM\cos\theta, \theta)]$. Let us note that the last equation of system (1.5) can be integrated independently of the rest. For known u, v, p, Φ the function S is defined uniquely by using the conditions (1.6). Consequently, we later study an independent subsystem of the first three equations in (1.5) by using the boundary conditions (1.6) that do not contain S. The transformed equations (1.5) have the form

$$\begin{gathered} \eta_\theta + i(\mu^2 - k^2)^{1/2}\eta_\mu = 0,\ \sigma_\theta - i(\mu^2 - k^2)^{1/2}\sigma_\mu = 0,\\ L_\theta + (M\mu\cos\theta - k)(M\sin\theta)^{-1}L_\mu = 0. \end{gathered} \quad (2.2)$$

An auxiliary complex variable associated with μ by the relationship $\mu = 2^{-1}k(\xi + \xi^{-1})$ is used below. The branch of the reverse mapping $\xi = \xi(\mu)$ is selected in such a way that the following domain D_θ belonging to the exterior of the unit circle $\mathbf{U}_3(\lambda, \mu, \theta)$ $\operatorname{Re}\mu > k_1 M\cos\theta$ $(k = k_1 + ik_2)$ belonging to the exterior of the unit circle $\beta = |\xi| \ge 1$ of the plane $\xi = \beta e^{i\varphi}$:

$$\text{a) } \varkappa \in (\pi/2, \pi]\colon \beta \geqslant 1, \quad (2.3)$$

$$2\pi - \varphi_1(\theta, \varkappa, \beta) - \varphi_2(\theta, \varkappa, \beta) < \varphi < 2\pi + \varphi_1(\theta, \varkappa, \beta) + \varphi_2(\theta, \varkappa, \beta);$$

corresponds to the analyticity domains of the functions

$$\text{b) } \varkappa \in [\pi, 3\pi/2]: \beta \geqslant 1,$$
$$\varphi_2(\theta, \varkappa, \beta) - \varphi_1(\theta, \varkappa, \beta) < \varphi < \varphi_2(\theta, \varkappa, \beta) + \varphi_1(\theta, \varkappa, \beta).$$

Here

$$\varphi_1(\theta, \varkappa, \beta) = \arccos[2M\beta \cos\theta \cos\varkappa((\beta^2-1)^2 + 4\beta^2\cos^2\varkappa)^{-1/2}];$$
$$\varphi_2(\theta, \varkappa, \beta) = \arccos[\cos\varkappa(\beta^2+1)((\beta^2-1)^2 + 4\beta^2\cos^2\varkappa)^{-1/2}]; \cos\varkappa = k_1|k|^{-1};$$
$$\sin\varkappa = k_2|k|^{-1}.$$

The branch $(\mu^2 - k^2)^{1/2}$ is selected so that $(\mu^2 - k^2)^{1/2} = 2^{-1}k(\xi - \xi^{-1})$. System (2.2) is integrated, its general solution has the form

$$\eta = f(\xi e^{-i\theta}), \ \sigma = l(\xi e^{i\theta}), \ L = W[(\mu M - k\cos\theta)/(M\sin\theta)], \tag{2.4}$$

where f, ℓ, and W are arbitrary analytic functions (the dependence on λ is not indicated in the notation). These functions should be determined by using the transformed boundary conditions (1.6). The function $\Phi_3(\lambda, \mu)$ associated with $\Phi(y, t) = \Phi(r(a^2 + 1 - M^2)^{-1/2}, t)$ by the same transformations that connect $U_3(\lambda, \mu, \theta)$ with $U(t, r, \theta)$ can be eliminated from the transformed relationships (1.6). The following conditions remain:

$$\begin{gathered}\theta = 0: \eta - \sigma = 0;\\ \theta = \theta_0: \eta = h\sigma + H, \ L = N\eta + Q\sigma + K.\end{gathered} \tag{2.5}$$

The coefficients h, H, N, Q, and K are given by the expressions

$$\begin{gathered}
h = -\frac{M^2R(1+\Delta)X^2 - (a^2+1-M^2)(1-\Delta)Y^2 + iM(1-M^2)^{1/2}(a^2+1-M^2)Y(\xi^2-1)}{M^2R(1+\Delta)X^2 - (a^2+1-M^2)(1-\Delta)Y^2 - iM(1-M^2)^{1/2}(a^2+1-M^2)Y(\xi^2-1)},\\
H = \frac{-kM(\xi-\xi^{-1})}{M^2R(1+\Delta)X^2 - (a^2+1-M^2)(1-\Delta)Y^2 - iM(1-M^2)^{1/2}(a^2+1-M^2)Y(\xi^2-1)} \times\\
\times\left[\frac{MX^2(1+\Delta)R}{2}F_{33} + \frac{M(a^2+1-M^2)^{1/2}}{1+a^2}XYF_{23} + \frac{a^2+1-M^2}{1+a^2}Y^2F_{13} + \right.\\
\left. + \frac{M^2(1-R)(M^2-1)(a^2+1-M^2)^{1/2}}{1+a^2}X\xi Z(\lambda)\right],\\
N = \frac{(M\operatorname{ch}(\nu - i\theta_0)-1)(M\operatorname{ch}(\nu+i\theta_0)-1)}{M\operatorname{sh}\nu(\operatorname{ch}\nu - M\cos\theta_0)} \times\\
\times\left[\frac{ia(1-M^2)^{1/2}\operatorname{sh}(\nu-i\theta_0) - \operatorname{ch}(\nu-i\theta_0)+M}{M\operatorname{ch}(\nu-i\theta_0)-1} + \frac{(1+a^2)(1-\Delta)}{2M}\right],\\
Q = \frac{(M\operatorname{ch}(\nu - i\theta_0)-1)(M\operatorname{ch}(\nu+i\theta_0)-1)}{M\operatorname{sh}\nu(\operatorname{ch}\nu - M\cos\theta_0)} \times\\
\times\left[\frac{-ia(1-M^2)^{1/2}\operatorname{sh}(\nu+i\theta_0) - \operatorname{ch}(\nu+i\theta_0)+M}{M\operatorname{ch}(\nu+i\theta_0)-1} + \frac{(1+a^2)(1-\Delta)}{2M}\right],\\
K = \frac{k(M\operatorname{ch}(\nu+i\theta_0)-1)(M\operatorname{ch}(\nu-i\theta_0)-1)}{M(\operatorname{ch}\nu - M\cos\theta_0)}F_{13}, \quad Z(\lambda) = \int_0^\infty e^{-\lambda t}x(t)\,dt.
\end{gathered} \tag{2.6}$$

Here $X = 2^{-1}(a^2 + 1 - M^2)^{1/2}(\xi^2 + 1) - aM\xi$; $Y = 2^{-1}aM(\xi^2 + 1) - \xi(a^2 + 1 - M^2)^{1/2}$, F_{i3} are obtained from $f_i(y, t) = f_i(r(a^2 + 1 - M^2)^{-1/2}, t)$ by the same transformations as U_3 from U; the complex variable ν is connected with μ by the relationship $\mu = k\cosh\nu$. The problem reduces to constructing the functions f, ℓ, and W satisfying conditions (2.5). Additional relationships and constraints on these functions, associated with the selection of the solutions of the original problem from a definite class, are formulated below. Methods for solving boundary-value problems of complex variable function theory are used in the construction of the Laplace transforms of the solution of the perturbations problem. The Riemann problem [12] occurs here as an auxiliary problem and then the solution of the Riemann problem is continued analytically into a broader domain. The question of the existence of a solution of the original problem is solved in calculating the index of the Riemann problem and in clarifying the existence conditions for analytic continuation.

After construction of the functions u, v, and p the functions $\Phi(y, t)$ yielding the location of the perturbed reflected front is found uniquely by using conditions (1.6).

It follows from (2.5) that $f(\xi) = \ell(\xi)$ and

$$f(\xi e^{-i\theta_0}) = h(\xi) f(\xi e^{i\theta_0}) + H(\xi) \tag{2.7}$$

(λ is omitted in the notation). By virtue of (2.5) the function W is expressed in terms of f. Therefore, it is sufficient to construct the function f. The analytic function P_3 is restored in terms of f by means of the formula

$$P_3 = \frac{f(\xi e^{i\theta}) + f(\xi e^{-i\theta})}{(\mu^2 - k^2)^{1/2}} = \frac{2(f(\xi e^{i\theta}) + f(\xi e^{-i\theta}))}{k(\xi - \xi^{-1})}, \tag{2.8}$$

while the functions U_3, V_3 are restored by formulas following from (2.1). Let us note that for $\mathrm{Re}\,\lambda > 0$ the branch point $\mu = -k$ of the mapping $(\mu^2 - k^2)^{1/2}$ is incident in the domain $\mathrm{Re}\,\mu > k_1 M \cos\theta$; consequently, it is necessary for the analyticity of $P_3(\lambda, \mu, \theta)$ in the mentioned domain that $f(\xi)$ satisfy the relationship

$$f(\xi) = -f(\xi^{-1}), \ |\xi| = 1 \tag{2.9}$$

on the section of the boundary $\beta = 1$ of the domain D_θ corresponding to the slit in the plane μ connecting the two branch points $\mu = \pm k$. There also results from the representation (2.8) that for P_3 to be analytic in D_θ the analyticity of f is necessary in the domain $D_\theta{}^1 = D_\theta \cup D_\theta{}^+ \cup D_\theta{}^-$, where $D_\theta{}^\pm \{\xi\!: \ \xi = \xi_0 e^{\pm i\theta}, \ \xi_0 \in D_\theta\}$. We are interested in the solution $p(t, r, \theta)$ bounded for $r = 0$. According to known properties of the Laplace transform

$$\lim_{\mu\to\infty, |\arg\mu| < \pi/2} (\mu P_2) = p|_{r=0},$$

Consequently, the solution of (2.7) and (2.9) should be sought in the class of functions analytic in $D_\theta{}^1$ and bounded as $|\xi| \to \infty$.

Construction of the function $f(\xi)$ satisfying relationships (2.7) and (2.9) in $D_{\theta_0}{}^1$ can reduce to the solution of a Riemann problem. It is sufficient to note that if f is known in the sector $S_{\theta_0} = \{\xi\!: \ \pi - \theta_0 \le \varphi \le \pi + \theta_0, \ \beta \ge 1\}$ ($S_{\theta_0} \subset D_{\theta_0}{}^1$), then outside the limits of the sector this function is continued analytically by using (2.7). Indeed, for $\pi - 3\theta_0 \le \varphi \le \pi - \theta_0$ the continuation is realized by means of the formula

$$f(\xi) = h(\xi e^{i\theta_0}) f(\xi e^{i2\theta_0}) + H(\xi e^{i\theta_0}), \tag{2.10}$$

and by the formula

$$f(\xi) = (f(\xi e^{-2i\theta_0}) - H(\xi e^{-i\theta_0}))(h(\xi e^{-i\theta_0}))^{-1}. \tag{2.11}$$

for $\Pi + 3\theta_0 < \varphi < \pi + \theta_0$.

Successive continuation by using (2.10) and (2.11) permits predetermination of $f(\xi)$ everywhere in $D_{\theta_0}{}^1$ if the values of f are known in S_{θ_0}. To construct f in the sector we map S_{θ_0} conformally on the plane with a slit along the positive real half-axis. The mapping has the form

$$\zeta = 2^{-1} - 4^{-1}\left(e^{-i\pi^2/\theta_0}\xi^{\pi/\theta_0} + e^{i\pi^2/\theta_0}\xi^{-\pi/\theta_0}\right)$$
$$\left(\xi^{\pi/\theta_0} = \beta^{\pi/\theta_0} e^{i\pi\varphi/\theta_0} \ \text{for} \ \xi = \beta e^{i\varphi}, \ \ \pi - \theta_0 \leqslant \varphi \leqslant \pi + \theta_0\right).$$

The Riemann problem occurs in the ζ plane: Find the function f analytic in a plane with the slit $\mathrm{Im}\,\zeta = 0$, $0 < \mathrm{Re}\,\zeta = \zeta_1 < \infty$ bounded at infinity and satisfying the conjugate condition for $\mathrm{Im}\,\zeta = 0$:

$$f^+(\zeta_1) = G(\zeta_1) f^-(\zeta_1) + m(\zeta_1) \qquad (0 < \zeta_1 < \infty),$$
$$G(\zeta_1) = \begin{cases} -1, & 0 < \zeta_1 < 1, \\ h\left(-(\zeta_1^{1/2} + (\zeta_1 - 1)^{1/2})^{2\theta_0/\pi}\right), & 1 < \zeta_1 < \infty, \end{cases}$$
$$m(\zeta_1) = \begin{cases} 0, & 0 < \zeta_1 < 1, \\ H\left(-(\zeta_1^{1/2} + (\zeta_1 - 1)^{1/2})^{2\theta_0/\pi}\right), & 1 < \zeta_1 < \infty \end{cases}$$

($f^{\pm}$ are limit values of f from the upper and lower half-planes). According to (2.6), for $1 < \zeta_1 < \infty$, $G(\zeta_1) = (A - iB)(A + iB)^{-1}$, where $A(1) = 0$, $B(1) = (R(1 + \Delta)(a^2 + 1 - M^2)^{-1} - (1 - \Delta)M^{-2})(aM + (a^2 + 1 - M^2)^{1/2})^2$, $A(\infty) = 2^{-1}a(1 - M^2)^{-1/2}$, $B(\infty) = 4^{-1}(R(1 + \Delta) - a^2(1 - \Delta))$. If the main flow with a weak reflected shock is considered, then $B(\infty) < 0$, $B(1) < 0$. For a flow with a strong reflected shock $B(\infty) > 0$; hence if conditions (1.3) are satisfied on the shock, then $B(1) < 0$ while if the main flow parameters satisfy the neutral stability conditions, $B(1) > 0$. The contour on which the conjugate condition is given is conveniently appended to the real axis I by setting $G = 1$ and $m = 0$ for $-\infty < \zeta_1 < 0$.

The question of solvability of the Riemann problem obtained with discontinuous coefficients is solved after evaluation of its index. The index of the problem in the class of functions bounded at points of discontinuity of the coefficients can be calculated from the formula [12]

$$\varkappa = (2\pi)^{-1}\Delta \arg G(\zeta_1)|_I,\ 0 < (2\pi)^{-1}\arg(G(c_i - 0)/G(c_i + 0)) < 1$$

[c_i are points of discontinuity of $G(\zeta_1)$]. The computation using this formula is realized sufficiently simply since changes in A and B along the contour as a function of the parameters of the problem are known. We consequently have: a) $\varkappa = -1$ for the main flow with a weak reflected shock; b) $\varkappa = -2$ for the main flow with a strong reflected shock on which the strong stability conditions are satisfied; c) $\varkappa = -1$ for the main flow with a strong reflected compression shock on which the neutral stability conditions are satisfied. According to the general theory, the Riemann problem is solvable uniquely in cases a and c. The solution exists in case b only when the right side $m(\zeta_1)$ satisfies a solvability condition of the form

$$\int_1^{\infty} m(\zeta_1)\,N(\zeta_1)\,d\zeta_1 = 0$$

($N(\zeta_1)$ is a function that is evaluated by means of the coefficients of the problem [12]). Therefore, in case b the perturbation problem is not solvable for general data. Explicit formulas by which f can be calculated in terms of G and m in cases a and c are presented in [12].

In the last two cases f must be continued from the sector in the domain $D_{\theta_0}{}^1$. The continuation will be analytic if the function $h(\xi)$ has no poles for $\operatorname{Im}\xi > 0$, while for the function $(h(\xi))^{-1}$ has no poles for $\operatorname{Im}\xi < 0$. According to (2.6), $h(\xi)$ is a rational function, the ratio of two fourth-degree polynomials. Moreover, it follows from (2.6) that $h(\xi) = (h(\xi^{-1}))^{-1}$. Consequently, if $\xi = a$ is a root of the polynomial in the denominator, then $\xi = a^{-1}$ is a root of the polynomial in the numerator. Poles of the function $h(\xi)$ are determined as roots of the equation

$$\begin{aligned} &M^2R(1 + \Delta)X^2 - (a^2 + 1 - M^2)Y^2(1 - \Delta) - \\ &- iM(1 - M^2)^{1/2}(a^2 + 1 - M^2)Y(\xi^2 - 1) = 0. \end{aligned} \tag{2.12}$$

<u>LEMMA</u>. If the main flow parameters satisfy strong stability conditions (1.3), then the polynomial (2.12) has no roots in the domain $\operatorname{Im}\xi > 0$. If the neutral stability conditions (1.4) are satisfied on the reflected shock for the main flow, then the polynomial (2.12) has the root $a = e^{i\gamma_1}$, $\theta_1 < \gamma_1 < \pi$ ($\cos\theta_1 = M\cos\theta_0$, $\sin\theta_1 = (a^2 + 1)^{1/2}\sin\theta_0$) on the boundary of the domain D_{θ_0}.

<u>Proof.</u> By using the rational fraction substitution

$$T = (\xi - \alpha)(\alpha\xi - 1)^{-1},\quad \alpha = e^{-i\theta_1} \tag{2.13}$$

Eq. (2.12) is reduced to the biquadratic

$$\left(1 + \frac{1-\Delta}{2M_n}\right)T^4 + \frac{1-\Delta}{M_n}\left(1 - \frac{2M_n^2R}{1-M_n^2}\frac{1+\Delta}{1-\Delta}\right)T^2 - 1 + \frac{1-\Delta}{2M_n} = 0, \tag{2.14}$$

analogous to (3.5) in [9]. It follows from the results in [9] that the roots T_i (2.14) satisfy the inequalities $|T_i| < 1$ for satisfaction of conditions (1.3). By virtue of (2.13) the appropriate roots a_i (2.12) satisfy the inequalities $\operatorname{Im} a_i < 0$.

If the inequalities (1.4) are satisfied for the main flow, then, as shown in [9], Eq. (2.14) has two roots within the unit circle ($\operatorname{Im} a_i < 0$ for appropriate a_i) and two real roots differing in sign outside the unit circle. By virtue of (2.13), the root of the polynomial (2.12) $a = e^{i\gamma_1}$, $\theta_1 < \gamma_1 < \pi$, belonging to the section $\beta = 1$ of the boundary of the domain D_{θ_0} corresponds to the real root $T_1 > 1$. The lemma is proved.

The results from the lemma, that $f(\xi)$ can be predetermined everywhere in $D_{\theta_0}{}^1$ for the main flow with a weak shock since no new singularities appear for analytic continuation by means of the relationships (2.10) and (2.11). It follows from the lemma that continuation of $f(\xi)$ certainly has poles on the boundary of the domain $D_{\theta_0}{}^1$ for a main flow with a strong reflected compression shock on which the neutral stability conditions are satisfied: $\beta = 1$, resulting in the appearance of poles of the function $P_3(\lambda, \mu, \theta_0)$ in the domain $\operatorname{Re}\mu > k_1 M \cos\theta_0$. This means that, in this case, the problem of constructing Laplace transforms of the solution of the perturbation problems is not solvable in the class of functions analytic in the domain $\operatorname{Re}\mu > k_1 M \cos\theta$. In particular, this leads to the absence of integrable square solutions in the variable r.

The fundamental functions describing the perturbed flow are restored by means of the Laplace transform found by application of an inverse transformation. Explicit expressions for the mentioned functions can be written by using representations of the Riemann problem solution. Estimates of the perturbations in terms of data of the problem are obtained by using estimates of the solution of the Riemann problem and the Parseval equality.

Uniqueness of the solution in a sufficiently broad class of functions (functions for which the applied integral transformations have meaning) follows from the single-valuedness of the algorithm for construction of the Laplace transforms.

In sum, correctness of the linear perturbation problem for stationary flow with a weak reflected compression shock and incorrectness of the corresponding problem for a main flow with a strong reflected shock have been established. By virtue of analogy of the formulations of perturbations problems, the same deductions are also obtained for problems of steady supersonic gas flow around an infinite wedge in the presence of attached compression shocks.

LITERATURE CITED

1. L. V. Ovsyannikov, Lectures on Gas Dynamic Principles [in Russian], Nauka, Moscow (1981).
2. G. F. Carrier, "On the stability of supersonic flows past a wedge," Quart. Appl. Math., 6, No. 4 (1949).
3. L. F. Henderson and J. D. Atkinson, "Multivalued solutions of steady-state supersonic flow. Part 1. Linear analysis," J. Fluid Mech., 78, Pt. 4 (1976).
4. A. M. Blokhin and E. I. Romenskii, "Stability of the stationary limit solution for flow around a circular cone," Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk, No. 13, Issue 3 (1978).
5. V. V. Rusanov and A. A. Sharakshane, "On the stability of flows around an infinite wedge or cone placed in a supersonic gas flow," Modern Problems of Mathematical Physics and Computational Mathematics [in Russian], Nauka, Moscow (1982).
6. M. D. Salas and B. D. Morgan, "Stability of shock waves attached to wedges and cones," AIAA J., 21, No. 12 (1983).
7. V. M. Teshukov, "On shock polars in a gas with general equations of state," Prikl. Mat. Mekh., 50, No. 1 (1986).
8. S. P. D'yakov, "On shock stability," Zh. Éksp. Teor. Fiz., 27, No. 3 (1954).
9. S. V. Iordanskii, "On stability of a plane stationary shock," Prikl. Mat. Mekh., 21, No. 4 (1957).
10. V. M. Teshukov, "On shock wave stability conditions," Dynamics of a Continuous Medium [in Russian], No. 76, Inst. Hydrodyn. Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1986).
11. A. M. Blokhin, Energy Integrals and Their Applications to Gas Dynamics Problems [in Russian], Nauka, Novosibirsk (1986).
12. N. I. Muskhelishvili, Singular Integral Equations [in Russian], Nauka, Moscow (1968).